



PAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	LEVEL: 7
COURSE CODE: MMO701S	COURSE NAME: MATHEMATICAL MODELLING 1
SESSION: JULY 2023	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 120 (to be converted to 100)

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS
<ol style="list-style-type: none">1. Attempt ALL the questions.2. All written work must be done in blue or black ink and sketches must be done in pencils.3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [25 MARKS]

- (a) Discuss mathematical modelling and its process with appropriate illustrated diagram. [9]
- (b) State the method of Conjecture in Mathematical modelling and employ the method to show that the solution of the dynamical system

$$a_{n+1} = ra_n + b, r \neq 1 \quad (1.1)$$

is given by

$$a_k = r^k c + \frac{b}{1-r} \quad (1.2)$$

for some c (which depends on the initial condition). [16]

QUESTION 2 [35 MARKS]

- (a) Consider an annuity where a savings account pays a monthly interest of 1.2% on the amount present and the investor is allowed to withdraw a fixed amount of N\$1200 monthly until the account is depleted. What is the solution of the dynamical system model for the annuity problem and how much of the initial investment will be needed to deplete the annuity in 22 years? State all appropriate theorems used in your solution. [24]
- (b) Given the following experimental data from a spring-mass system:

Mass	50	100	150	200	250
Elongation	1.200	1.650	2.000	3.150	4.200

Formulate two different models that estimate the proportionality of the elongation to the mass, clearly showing how your proportionality constant is obtained for each model. [11]

QUESTION 3 [30 MARKS]

- (a) Construct natural cubic splines that pass through the following data points.

x_i	0	1	2
y_i	0	5	8

[15]

- (b) Consider the following table of data:

x	1	2.3	3.5	4.5	6.5	7.0
y	3.5	3.2	5.5	6.2	4.5	7.5

- (i) Estimate the coefficients of the straight line $y = ax + b$ such that the sum of the squared deviations of the data points and the line is minimised. [5]

(ii) If the largest absolute deviations for the Chebyshev's criterion and that of the Least Squares criterion are given respectively by c_{max} and d_{max} , define them and then compute their values including their least bound D to express their relationship for the above data and the model line. [10]

QUESTION 4 [30 MARKS]

- (a) Suppose a certain drug is effective in treating a disease if the concentration remains above 120 mg/L. The initial concentration is 645 mg/L. It is known from laboratory experiments that the drug decays at the rate of 25% of the amount present each hour.
- (i) Formulate a model representing the concentration at each hour. [2]
- (ii) Build a table of values (answer correct to 2 decimal places) and determine when the concentration reaches 120 mg/L. [12]

- (b) Consider the following table showing the experimental data of the growth of a micro-organism.

n	0	1	2	3	4	5	6
y_n	8.2	15.3	29.2	45.5	71.1	120.1	174.6
Δy_n	8.7	11.7	16.3	23.9	52	55.5	85.6

where n is the time in days and y_n is the observed organism biomass.

- (i) Construct a linear model for the above organism growth and show that the model predicts an increasing population without limit.
- (ii) Assume that contrary to your model prediction in (i), there is a maximum population of 665. Hence formulate a nonlinear dynamical system model for the organism using your constant obtained from an appropriate ratio similar to the example given in class, for $n = 3$ in the above data. [16]

END OF QUESTION PAPER

TOTAL MARKS = 120